MENIIT
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JEE MAINS-2013

25-04-2013 (Online-4)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-PHYSICS

1. In an experiment, a small steel ball falls through a liquid at a constant speed of 10 cm/s. If the steel ball is pulled upward with a force equal to twice its effective weight, how fast will it move upward ?

(1) Zero (2*) 10 cm/s (3) 20 cm/s (4) 5 cm/s

Sol. Weight of the body

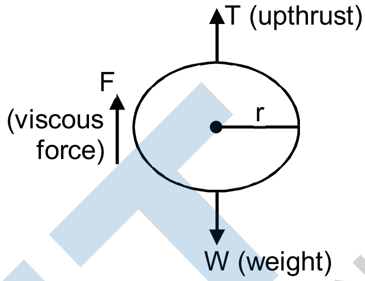
$$W = mg = \frac{4}{3}\pi r^3 \rho g$$

$$T = \frac{4}{3}\pi r^3 \sigma g$$

and F = $6\pi\eta v r$

when the body attains terminal velocity net force acting on the body is zero. i.e.,

W – T – F = 0



And terminal velocity $v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

As in case of upward motion upward force is twice its effective weight, therefore, it will move with same speed 10 cm/s

2. A printed page is pressed by a glass of water. The refractive index of the glass and water is 1.5 and 1.33, respectively. If the thickness of the bottom of glass is 1 cm and depth of water is 5 cm, how much the page will appear to be shifted if viewed from the top ?

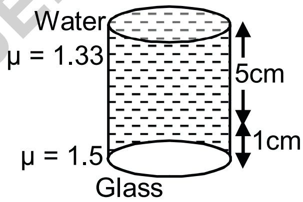
(1*) 1.3533 cm (2) 1.033 cm (3) 3.581 cm (4) 1.90 cm

Sol. Real depth = 5 cm + 1 cm = 6 cm

Apparent depth $= \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + ...$

$$= \frac{5}{1.33} + \frac{1}{1.5}$$

= 3.8 + 0.7 ≈ 4.5 cm



$\therefore$ Shift = 6 cm – 4.5 cm ≅ 4.5 cm

3. A ring of mass M and radius R is rotating about its axis with angular velocity ω. Two identical bodies each of mass m are now gently attached at the two ends of a diameter of the ring. Because of this, the kinetic energy loss will be:

(1) $\frac{(M+m)M}{(M+2m)}\omega^2R^2$ (2) $\frac{Mm}{(M+m)}\omega^2R^2$ (3*) $\frac{Mm}{(M+2m)}\omega^2R^2$ (4) $\frac{m(M+2m)}{(M+2m)}\omega^2R^2$

Sol. Kinetic energy$_{(rotational)}$ $K_R = \frac{1}{2}I\omega^2$

Kinetic energy$_{Translational}$ $K_r = \frac{1}{2}mv^2$ (v = Rω)

M.I.$_{(initial)}$ $I_{ring} = MR^2$; $w_{initial} = \omega$

M.I.$_{(new)}$ $I'_{(system)} = MR^2$; $2mR^2$

$$\omega'_{(system)} = \frac{M\omega}{M+2m}$$

Solving we get loss in K.E.

$$= \frac{Mm}{(M+2m)}\omega^2R^2$$

4. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. It will emit :

(A*) 2 lines in the Lyman series and 1 line in the Balmar series

(B) 3 lines in the Balmer series

(C) 1 line in the Lyman series and 2 lines in the Balmar series

(D) 3 lines in the Lyman series

Sol. $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.62\times10^{-34}\times3\times10^{8}}{12.5\times1.6\times10^{-19}} = 993\text{Å}$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

(where Rydberg constant, R = 1 = 1.097×10^7)

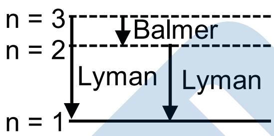
or, $\frac{1}{993\times10^{-10}} = 1.097\times10^{7}\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$

Solving we get $n_2 = 3$

Spectral lines

Total liens in Lyman series for $n_1 = 1$, $n_2 = 2$ and $n_1 = 1$, $n_2 = 3$

and one in Balmer series for $n_1 = 2$, $n_2 = 3$

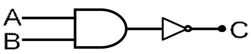


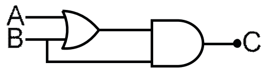
5. The source that illuminates the double-slit in 'double-slit interference experiment' emits two distinct monochromatic waves of wave length 500 nm and 600 nm, each of them producing its own pattern on the screen. At the central point of the pattern when path difference is zero, maxima of both the patterns coincide and the resulting interference pattern is most distinct at the region of zero path difference. But as one moves out of this central region, the two fringe systems are gradually out of step such that maximum due to one wave length coincides with the minimum due to the other and the combined fringe systems are gradually out of step such that maximum due to one wave length coincides with the minimum due to the other and the combined fringe system becomes completely indistinct. This may happen when path difference in nm is :

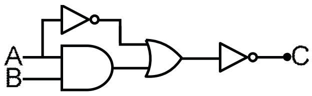
(1) 1000 (2) 2000 (3) 3000 (4*) 1500

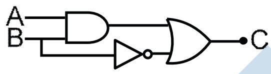
5. Which of the following circuits correctly represents the following truth table ?

A	B	C
0	0	0
0	1	0
1	0	1
1	1	0

(1) 

(2) 

(3*) 

(4) 

Sol. For circuit 1

$A \cdot B = \overline{Y + \bar{A}} = C$

A	B	$\overline{Y+\bar{A}} = C$		
0	0	0	1	0
0	1	0	1	0
1	0	0	0	1
1	1	1	0	0

7. A metal sample carrying a current along X-axis with density J_x is subjected to a magnetic field B_z (along z-axis). The electric field E_y developed along Y-axis is directly proportional to J_x as well as B_z. The constant of proportionality has SI unit.

(1*) $\frac{m^3}{A_s}$ (2) $\frac{m^2}{A_s}$ (3) $\frac{A_s}{m^3}$ (4) $\frac{m^2}{A}$

Sol. According to question

$E_y \propto J_x B_z$

$\therefore$ Constant of proportionality

$$K = \frac{E_y}{B_z J_x} = \frac{C}{J_x} = \frac{m^3}{As}$$

[As $\frac{E}{B} = C$ (speed of light) and $J = \frac{I}{\text{Area}}$]

8. In the isothermal expansion of 10 g of gas from volume V to 2V the work done by the gas is 575 J. What is the root mean square speed of the molecules of the gas at that temperature ?

(1*) 499 m/s (2) 532 m/s (3) 398 m/s (4) 520 m/s

Sol. $v_{rms} = \sqrt{\frac{3\rho v}{\text{mass of the gas}}}$

9. A mass of 50 g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from 30° C to 25°C. A mass of 100 g of another liquid in an identical vessel with identical surroundings takes the same time to cool from 30°C to 25°C. The specific heat of the liquid is :

(1) 2.0 kcal/kg (2) 7 kcal/kg (3) 3 kcal/kg (4*) 0.5 kcal/kg

Sol. As the surrounding is identical, vessel is identical time taken to cool both water and liquid (from 30ºC to 25ºC) is same 2 minutes, therefore

$$\left(\frac{dQ}{dt}\right)_{water} = \left(\frac{dQ}{dt}\right)_{liquid}$$

or, $\frac{(m_w C_w + W)\Delta T}{t} = \frac{(m_\ell C_\ell + W)\Delta T}{t}$

(W = water equivalent of the vessel)

or, $m_w C_w = m_\ell C_\ell$

$\therefore$ Specific heat of liquid, $C_\ell = \frac{m_w C_w}{m_\ell}$

$= \frac{50 \times 1}{100} = 0.5 \text{kcal/kg}$

10. This question has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1 : No work is required to be done to move a test charge between any two points on an equipotential surface.

Statement 2 : Electric lines of force at the equipotential surfaces are mutually perpendicular to each other .

(1) Statement is true, Statement 2 is true, Statement-2 is not the correct explanation of Statement-2

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is the correct explanation of Statement-2.

(4*) Statement 1 is true, Statement-2 is false

Sol. The work done is moving a charge along an equipotential surface is always zero.

The direction of electric field is perpendicular to the equipotential surface or lines.

11. In a transverse wave the distance between a crest and neighbouring through at the same instant is 4.0 cm and the distance between a crest and through at the same place is 1.0 cm. The next crest appears at the same place after a time interval of 0.4 s. The maximum speed of the vibrating particles in the medium is :

(1) $\frac{\pi}{2}$cm/s (2) $\frac{3\pi}{2}$cm/s (3*) $\frac{5\pi}{2}$cm/s (4) 2π cm/s

12. Which of the following modulated signal has the best noise-tolerance ?

(1*) Short-wave (2) Medium-wave

(3) long-wave (4) amplitude-modulated

Sol. Short-wave has the best noise tolerance.

13. A series LR circuit is connected to an ac source of frequency ω and the inductive reactance is equal to 2R. A capacitance of capacitive reactance equal to R is added in series with L and R. The ratio of the new power factor to the old one is :

(A) $\sqrt{\frac{3}{2}}$ (B) $\sqrt{\frac{2}{5}}$ (C*) $\sqrt{\frac{5}{2}}$ (D) $\sqrt{\frac{2}{3}}$

Sol. Power factor(old)

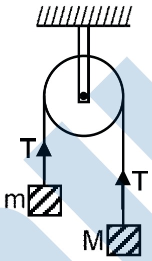
$$= \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + (2R)^2}} = \frac{R}{\sqrt{5}R}$$

Power factor(new)

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + (2R - R)^2}} = \frac{R}{\sqrt{2}R}$$

$$\therefore \frac{\text{New power factor}}{\text{Old power factor}} = \frac{\frac{R}{\sqrt{2}R}}{\frac{R}{\sqrt{5}R}} = \sqrt{\frac{5}{2}}$$

14. Two blocks of masses m and M are connected by means of a metal wire of cross-sectional area A passing over a frictionless fixed pulley as shown in the figure. The system is then released. If M = 2m, then the stress produced in the wire is :



(1) $\frac{2mg}{3A}$ (2) $\frac{3mg}{4A}$

(3) $\frac{mg}{A}$ (4*) $\frac{4mg}{3A}$

Sol. Tension in the wire, $T = \left(\frac{2mM}{m+M}\right)g$

$$\text{Stress} = \frac{\text{Force / Tension}}{\text{Area}} = \frac{2mM}{A(m+M)}g$$

$$= \frac{2(m+2m)g}{A(m+2m)} \quad (M = 2m \text{ given})$$

$$\frac{4m^2}{3mA}g = \frac{4mg}{3A}$$

15. A wind - powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be most likely proportional to :

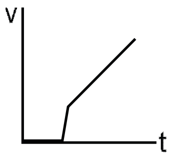
(1*) v^3 (2) v^2 (3) v^4 (4) v

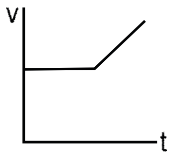
16. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, form a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a downward push and released, it starts oscillating vertically with a small amplitude. The time period T of the oscillations of the cylinder will be :

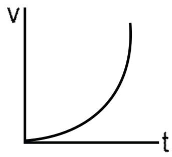
(1) $2\pi\sqrt{\frac{M}{k}}$ (2*) Smaller then $2\pi\left[\frac{M}{(a + A\sigma g)}\right]^{1/2}$

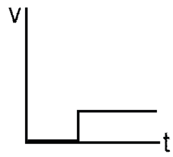
(3) $2\pi\left[\frac{M}{(a + A\sigma g)}\right]^{1/2}$ (4) Larger then $2\pi\left[\frac{M}{(a + A\sigma g)}\right]^{1/2}$

17. A block is placed on a rough horizontal plane. A time dependent horizontal force F = kt acts on the block, where k is a positive constant. The acceleration - time graph of the block is :

(1*) 

(2) 

(3) 

(4) 

Sol. Graph (a) correctly depicts the acceleration-time graph of the block.

18. The maximum range of a bullet fired from a toy pistol mounted on a car at rest is R_0 = 40 m. What will be the acute angle of inclination of the pistol for maximum range when the car i moving in the direction of firing with uniform velocity v = 20 m/s, on a horizontal surface ? (g = 10 m/s^2)

(1) 75° (2) 30° (3) 45° (4*) 60°

19. When resonance is produced in a series LCR circuit, then which of the following is not correct ?

(1*) Impedance of circuit is maximum

(2) If R is reduced, the voltage across capacitor will increase

(3) Current in the circuit is in phase with the applied voltage

(4) Inductive and capacitive reactances are equal

Sol. Impedance (Z) of the series LCR circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance, $X_L = X_C$

Therefore, $Z_{minimum} = R$

20. The earth's magnetic field lines resemble that of a dipole at the centre of the earth. If the magnetic moment of this dipole is close to 8 × 10^{22} Am^2, the value of earth's magnetic field near the equator is close to (radius of the earth = 6.4 × 10^6 m)

(A) 0.32 Gauss (B) 1.2 Gauss (C) 1.8 Gauss (D*) 0.6 Gauss

Sol. Given M = 8 × 10^{22} Am^2

$d = R_e = 6.4 \times 10^6$ m

Earth's magnetic field, $B = \frac{\mu_0}{4\pi}, \frac{2M}{d^3}$

$$= \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 8 \times 10^{22}}{(6.4 \times 10^6)^3}$$

$\cong$ 0.6 Gauss

21. One of the two small circular coils, (none of them having any self- inductance) is suspended with a V-shaped copper wire, with plane horizontal. The other coil is placed just below the first one with plane horizontal. Both the coils are connected in series with a dc supply. The coils are found to attract each other with a force. Which one of the following statements is incorrect ?

(1) Force is proportional to d^{-1}

(2) Coils will attract each other, even if the supply is an ac source

(3) Both the coils carry currents in the same direction

(4*) Force is proportional to d^{-2}

Sol. As the force between the two coils is attractive hence currents in the coil must be in the same direction and the force, $F \propto \frac{I_1 I_2}{2\pi d}$

where I_1 and I_2 are the currents flowing through the coils.

22. The gravitational field in a region is given by:

$\vec{E} = (5N/kg)\hat{i} + (12N/kg)\hat{j}$

If the potential at the origin is taken to be zero, then the ratio of the potential at the points (12m, 0) and (0, 5m) is :

(A*) 1 (B) $\frac{144}{25}$ (C) Zero (D) $\frac{25}{144}$

Sol. From question, $E_x = 5$ N/kg and $E_y = 12$ N/kg

Gravitational potential = Gravitational field × distance

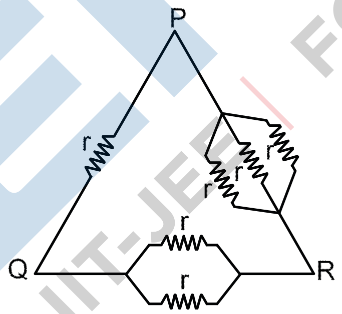
$\therefore$ $V_{(12m, 0)} = E_x \times 12$ J/kg

and $V_{(0, 5m)} = E_y \times 5$ J/kg

(Given : potential at the origin is zero)

$$\therefore \frac{V_{(12m,0)}}{V_{(0,5m)}} = \frac{E_x \times 12}{E_y \times 5} = \frac{5 \times 12}{12 \times 5} = 1$$

23. Six equal resistances are connected between points P, Q and R as shown in figure. Then net resistance will be maximum between :



(1) Any two points (2) Q and R (3*) P and Q (4) P and R

Sol. Resistance between P and Q

$$r_{PQ} = r_{11}\left(\frac{r}{3} + \frac{r}{2}\right) = \frac{r \times \frac{5}{6}r}{r + \frac{5}{6}r} = \frac{5}{11}r$$

Resistance between Q and R

$$r_{QR} = \frac{r}{2}11\left(r + \frac{r}{3}\right) = \frac{\frac{r}{2} \times \frac{4}{3}r}{\frac{r}{2} + \frac{4}{3}r} = \frac{4}{11}r$$

Resistance between P and R

$$r_{PR} = \frac{r}{3} 11 \left(\frac{r}{2} + r\right) = \frac{\frac{r}{3} \times \frac{3}{2} r}{\frac{r}{3} + \frac{3}{2} r} = \frac{3}{11} r$$

Hence, it is clear that r_{PQ} is maximum

24. The surface charge density of a thin charged disc of radius R is σ. The value of the electric field at the centre of the disc is $\frac{\sigma}{2 \epsilon_0}$. With respect to the field at the centre, the electric field along the axis at a distance R from the centre of the disc.

(A) reduces by 29.3%
(B) reduces by 9.7%
(C*) reduces by 70.7%
(D) reduces by 14.6%

Sol. Electric field intensity at the centre of the disc.

$$E = \frac{\sigma}{2 \epsilon_0} \quad \text{(given)}$$

Electric field along the axis at any distance x from the centre of the disc

$$E' = \frac{\sigma}{2 \epsilon_0} \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

From question, x = R (radius of disc)

$$\therefore E' = \frac{\sigma}{2 \epsilon_0} \left(1 - \frac{R}{\sqrt{R^2 + R^2}}\right)$$

$$= \frac{\sigma}{2 \epsilon_0} \left(\frac{\sqrt{2} R - R}{\sqrt{2} R}\right)$$

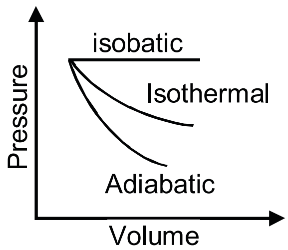
$$= \frac{4}{14} E$$

$\therefore$ % reduction in the value of electric field

$$= \frac{\left(E - \frac{4}{14} E\right) \times 100}{E} = \frac{1000}{14} \% = 70.7\%$$

25. A sample of gas expands from V_1 to V_2. In which of the following, the work done will be greatest ?

(1*) Isobaric process
(2) Same in all processes
(3) Isothermal process
(4) Adiabatic process



Sol. Work done = Area bounded by PV graph and volume axis

Among the three process, bounded area by PV graph and volume axis is greatest for isobaric process, hence work done is greatest for isobaric process.

26. A copper ball of radius 1 cm and work function 4.47 eV is irradiated with ultraviolet radiation of wavelength 2500 Å. The effect of irradiation results in the emission of electrons from the ball. Further the ball will acquire charge and due to this there will be a finite value of the potential on the ball. The charge acquired by the ball is :

(1) 4.5×10^{-12} C (2) 2.5×10^{-11} C (3) 7.5×10^{-11} C (4*) 5.5×10^{-13} C

27. A thin glass plate of thickness $\frac{2500}{3}\lambda$ (λ is wavelength of light used) and refractive index $\mu = 1.5$ is inserted between one of the slits and the screen in Young's double slit experiment. At a point on the screen equidistant from the slits, the ratio of the intensities before and after the introduction of the glass plate is :

(1) 2 : 1 (2*) 4 : 1 (3) 4 : 3 (4) 1 : 4

28. A parallel plate capacitor having a separation between the plates d, plate area a and material with dielectric constant K has capacitance C_0. Now one-third of the materials replaced by another material with dielectric constant 2 K, so that effectively there are two capacitors one with area $\frac{1}{3}A$, dielectric constant 2 K and another with area $\frac{2}{3}A$ and dielectric constant K. If the capacitance of this new capacitor is C then $\frac{C}{C_0}$ is :

(1) 1 (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4*) $\frac{4}{3}$

Sol. $C_0 = \frac{k \epsilon_0 A}{d}$

$$C = \frac{k \epsilon_0 2}{3d} + \frac{2k \epsilon_0 A}{3d} = \frac{4}{3}\frac{k \epsilon_0 A}{d}$$

$$\therefore \frac{C}{C_0} = \frac{\frac{4}{3}\frac{k \epsilon_0 A}{d}}{\frac{k \epsilon_0 A}{d}} = \frac{4}{3}$$

29. This question has Statement-1 and Statement-2. Of the four choices given after the Statement, choose the one that best describes the two Statements.

Statement-1: Out of radio waves and microwaves, the radio waves undergo more diffraction.

Statement-2 : Radio waves have greater frequency compared to microwaves.

(1*) Statement-1 is true, Statement-2 is false

(2) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1

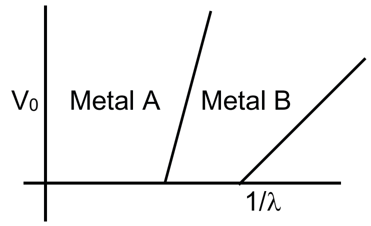
(3) Statement-1 is true, Statement-2 is true but Statement-2 is not the correct explanation of Statement-1

(4) Statement-1 is false, Statement-2 is true

Sol. Wavelength of radio waves is greater than microwaves hence frequency of radio waves is less than microwaves.

The degree of diffraction is greater whose wavelength is greater.

30. In an experiment on photoelectric effect, a student plots stopping potential V_0 against reciprocal of the wavelength λ of the incident light for two different metals A and B. These are shown in the figure. Looking at the graphs, you can most appropriately say that :



(A) for light of a certain frequency falling on both the metals, the kinetic energy of ejected photoelectrons from A will be greater than that from B

(B) Work function of metal B is greater than that of metal A

(C) Work function of metal A is greater than that of metal B

(D*) Students data is not correct

Sol. $\frac{hc}{\lambda} - \phi = eV_0$

$v_0 = \frac{hc}{e\lambda} - \frac{\phi}{e}$

For metal A

$\frac{\phi A}{hc} = \frac{1}{\lambda}$

For metal B

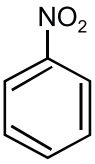
$\frac{\phi B}{hc} = \frac{1}{\lambda}$

As the value of $\frac{1}{\lambda}$ (increasing and decreasing) is not specified hence we cannot say that which metal has comparatively greater or lesser work function (ϕ).

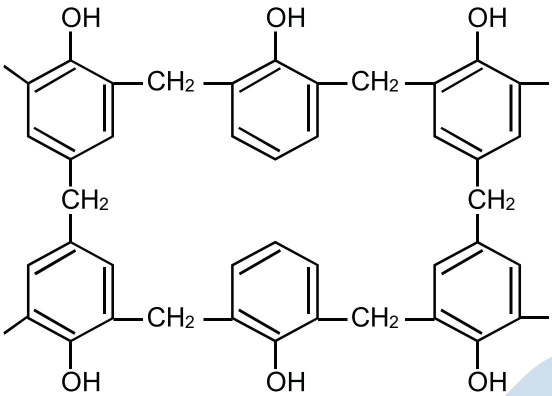
PART-B-CHEMISTRY

31. Bakelite is obtained from phenol by reacting with :

(1) Acetal (2) CH_3CHO (3*) HCHO (4) Chlorobenzene

Sol. n  + n HCHO $\xrightarrow{OH^-}$

Phenol Formaldehyde



Bakelite

32. A solution of copper sulphate ($CuSO_4$) is electrolysed for 10 minutes with a current of 1.5 amperes. The mass of copper deposited at the cathode (at. mass of Cu = 63u) is:

(1) 0.3928 g (2) 0.2398 g (3*) 0.2938 g (4) 0.3892 g

Sol. W = Zit

where Z = electrochemical equivalent

Eq. wt. of copper $= \frac{63}{2} = 31.5$

$Z = \frac{31.5}{96500}$

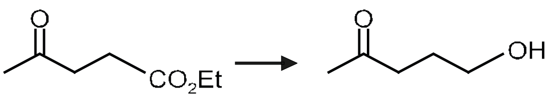
$W W = Zit = \frac{31.5}{96500} \times 1.5 \times 10 \times 60 = 0.2938g$

33. In which of the following exothermic reactions, the heat liberated per mole is the highest ?

(1) $SrO + H_2O \rightarrow Sr(OH)_2$ (2) $MgO + H_2O \rightarrow Mg(OH)_2$

(3) $BaO + H_2O \rightarrow Ba(OH)_2$ (4*) $CaO + H_2O \rightarrow Ca(OH)_2$

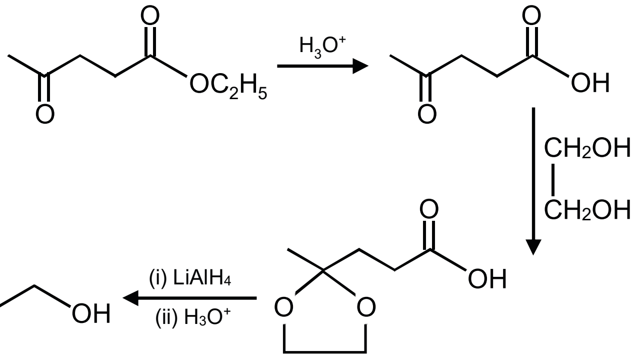
34. Which of the following reagent(s) used for the conversion



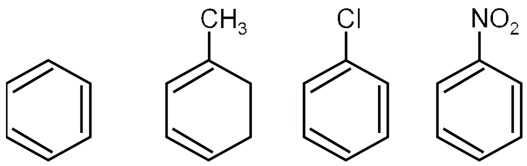
(1) glycol / NaH / H_3O^+ (2*) glycol / $LiAlH_4$ / H_3O^+

(3) $LiAlH_4$ (4) $NaBH_4$

Sol.



35. Given

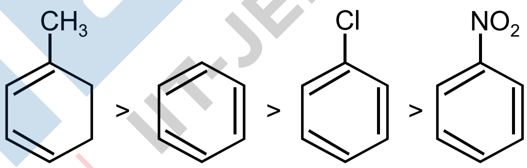


(a) (b) (c) (d)

In the above compounds correct order of reactivity in electrophilic substitution reactions will be :

(1) d > c > b > a (2*) b > a > c > d (3) a > b > c > d (4) b > c > a > d

Sol. –Cl and $-CH_3$ groups are o and p directing. They are electron releasing due to +E and +M effects. Further since such groups increase electron density in the nucleus, they facilitate further electrophilic substitution and hence known as activating group. The activating effect of these groups is in order of $-CH_3 > -X$ but chlorine exceptionally deactive the ring due to strong –I effect. Hence, its is difficult to carry out substitution in chlorobenzene than in benzene. Further $-NO_2$ is a deactivating group hence deactivates the benzene nucleus, i.e. hinder the further substitution. Thus nitrobenzene undergo electrophilic substitution. Thus nitrobenzene undergo electrophilic substitution with a great difficulty hence the correct order will be.



(b) (a) (c) (d)

36. What would be the pH of a solution obtained by mixing 5g of acetic acid and 7.5 g of sodium acetate and making the volume equal to 500 mL ?

(Ka = 1.75×10^{-5} , pKa = 4.76)

(1) pH = 4.70

(2*) 4.76 < pH < 5.0

(3) pH of solution will be equal to pH of acetic acid

(4) pH < 4.70

Sol. $pH = pK_a + \log\frac{[salt]}{[acid]}$

$$= 4.76 + \log\frac{\frac{7.5}{500}}{\frac{5}{500}} = 4.7 + \log 1.5 = 4.87$$

Hence correct answer is 4.76 < pH < 5.0

37. 10 mL of 2(M) NaOH solution is added to 200 mL of 0.5 (M) of NaOH solution. What is the final concentration ?

(1) 5.7 (M) (2*) 0.57 (M) (3) 11.4 (M) (4) 1.14 (M)

Sol. From molarity equation

$M_1V_2 + M_2V_2 = MV_{(total)}$

$$2 \times \frac{2}{1000} + 0.5 \times \frac{200}{1000} = M \times \frac{210}{1000}$$

120 = M × 210

$$M = \frac{120}{210} = 0.57M$$

38. The catenation tendency of C, Si and Ge is in the order Ge < Si < C. The bond energies (in kJ mol^{-1}) of C–C, Si – Si and Ge–Ge bonds are respectively :

(1*) 348, 297, 260 (2) 297, 348, 260 (3) 260, 297, 248 (4) 348, 260, 297

Sol. The linking of identical atoms with each other to form long chains is called catenation. However, this property decreases from carbon to lead. Decrease of this property is associated with M-M bond energy which decreases from carbon to lead.

39. Given

(a) n = 5, m_ℓ = + 1

(b) n = 2, ℓ = 1, m_ℓ = –1, m_s = –1/2

The maximum number of electron(s) in an atom that can have the quantum numbers as given in (a) and (b) are respectively ?

(1) 4 and 1 (2) 25 and 1 (3) 2 and 4 (4*) 8 and 1

Sol. (i) n = 5 means ℓ = 0, 1, 2, 3, 4

since m = + 1

hence total no. of electrons will be = 0 (from s) + 2 (from p) + 2 (from d) + 2 (from f) + 2 (from g)

= 0 + 2 + 2 + 2 + 2 = 8

(ii) n = 2, ℓ = l, m_ℓ = – 1, m_s = – 1/2 represent 2p orbital with one electron

40. Copper crystallises in fcc with a unit length of 361 pm. What is the radius of copper atom ?

(1) 181 pm (2) 157 pm (3*) 128 pm (4) 108 pm

Sol. For FCC,

$$r = \frac{\sqrt{2}a}{4} = \frac{a}{2\sqrt{2}} = 0.3535a$$

given a = 361 pm

r = 0.3535 × 361

= 128 pm

41. Which one of the following is the wrong assumption of kinetic theory of gases?

(1*) All the molecules move in straight line between collision and with same velocity

(2) Momentum and energy always remain conserved.

(3) Molecules are separated by great distances compared to their sizes.

(4) Pressure is the result of elastic collision of molecules with the container's wall.

Sol. Molecules move very fast in all directions in a straight line by colliding with each other but with different velocity.

42. Which one of the following cannot function as an oxidising agent?

(1*) I^- (2) $Cr_2O_7^{2-}$ (3) NO_3^- (aq) (4) S(s)

Sol. If an electronegative element is in its lowest possible oxidation state in a compound or in free state. It can function as a powerful e.g. I^-

43. Which of the following statement is not correct?

(1) Cellulose is a linear polymer of β-glucose.

(2) All proteins are polymers of α-amino acids.

(3*) Glycogen is the food reserve of plants

(4) Amylopectin is a branched polymer of α-glucose.

Sol. Glycogen is called animal starch and is found in all animal cells. It constitutes the reserve food material.

44. A radioactive isotope having a half-life period of 3 days was received after 12 days. If 3 g of the isotope is left in the container, what would be the initial mass of the isotope?

(1) 24 g (2) 12 g (3) 36 g (4*) 48 g

Sol. Given $t_{1/2} = 3$

Total time T = 12

No. of half lives $(n) = \frac{12}{3} = 4$

$$\left(\frac{1}{2}\right)^n = \frac{N}{N_0}$$

$$\therefore \left(\frac{1}{2}\right)^4 = \frac{3}{N_0}$$

$$\frac{3}{N_0} = \frac{1}{16}$$

$N_0 = 48$ g

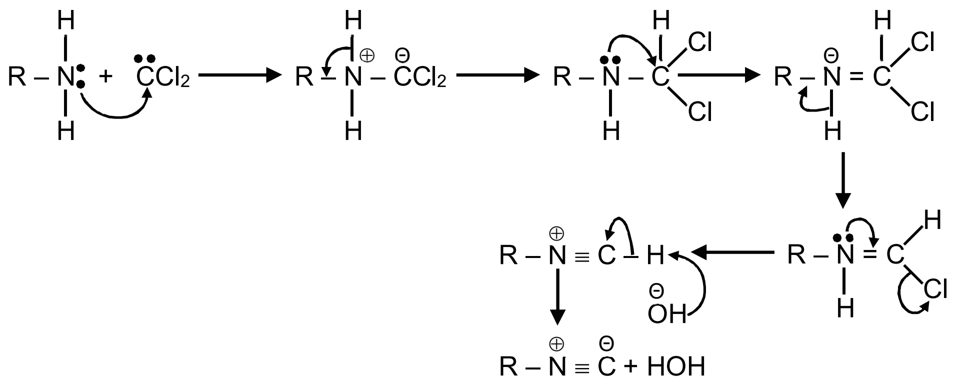
45. Carbylamine forms from aliphatic or aromatic primary amine via which of the following intermediates?

(1*) Carbene (3) Carbon radical (3) Carbocation (4) Carbanion

Sol. $RNH_2 + CHCl_3 + 3KOH \longrightarrow RNC + 3KCl + 3H_2O$

Mechanism

$$CHCl_3 \xrightarrow[\alpha-\text{elimination}]{OH^-/\Delta} \underset{\text{Dichlorocarbene}}{\ddot{C}Cl_2}$$



46. In which of the following sets, all the given species are isostructural?

(1) BF_3, NF_3, PF_3, AlF_3
(2*) BF_4^-, CCl_4, NH_4^+, PCl_4^+
(3) CO_2, NO_2, ClO_2, SiO_2
(4) PCl_3, $AlCl_3$, BCl_3, $SbCl_3$

Sol. All have tetrahedral structure.

47. In nucleophilic substitution reaction, order of halogens as incoming (attacking) nucleophile is :

$I^- > Br^- > Cl^-$

The order of halogens as departing nucleophile should be :

(1*) $I^- > Br^- > Cl^-$
(2) $Cl^- > I^- > Br^-$
(3) $Br^- > I^- > Cl^-$
(3) $Cl^- > Br^- > I^-$

Sol. Since the leaving group breaks away as a base, it is easier to displace weaker bases as compared to stronger bases. Thus less basic the substituent, the more easily it is displaced.

Thus the basic strength of the given groups is in order.

$I^- > Br^- > Cl^-$

48. How many grams of methyl alcohol should be added to 10 litre tank of water to prevent its freezing at 268 K ? [K_f for water is 1.86 K kg mol^{-1}]

(1) 899.04 g
(2) 886.02 g
(3*) 868.06 g
(4) 880.07 g

Sol. Crystal field splitting depends upon the nature of ligand. The nature of ligand Δ decreases as shown below $C_2H_4^{--} < H_2O < NH_3 < CN^-$

hence the crystal field splitting will be maximum for $[Co(CN)_6]^{3-}$

49. Which one of the following arrangement represents the correct order of the proton affinity of the given species:

(1) $NH_2^- < HS^- < I^- < F^-$
(2*) $I^- < F^- < HS^- < NH_2^-$
(3) $HS^- < NH_2^- < F^- < I^-$
(4) $F^- < I^- < NH_2^- < HS^-$

Sol. The species with the greatest proton affinity will be the strongest base, and its conjugate acid will be the weakest acid. The weakest acid will have the smallest value of K_a. Since HI is stronger acid than HF which is a stronger acid than H_2S, a partial order of proton affinity is

$I^- < F^- < HS^-$

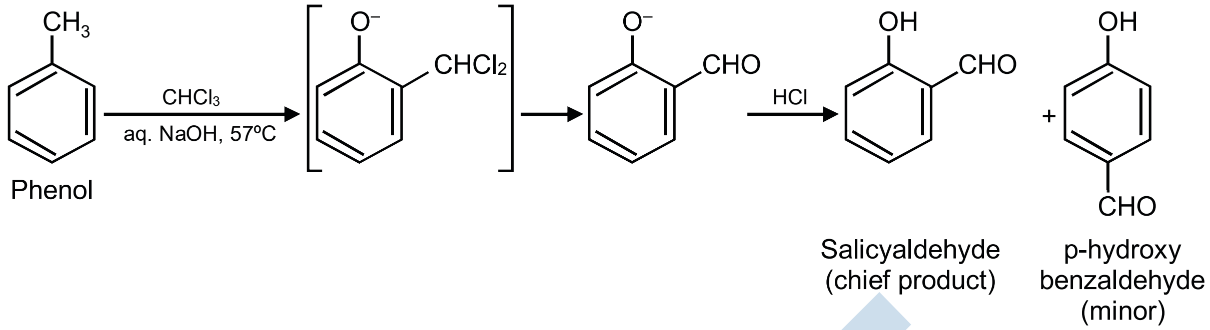
Since NH_3 is a very weak acid, NH_2^- must be a very strong base. Therefore the correct order of proton affinity is

$I^- < F^- < HS^- < NH_2^-$

50. Phenol on heating with $CHCl_3$ and NaOH gives salicylaldehyde. The reaction is called:

(1) Claisen reaction
(2*) Reimer – Tiemann reaction
(3) Hell – Volhard – Zelinsky reaction
(4) Cannizzaro's reaction

Sol.



51. Given that :

(i) $\Delta_f H°$ of N_2O is 83 kJ mol^{-1}

(ii) Bond energies of N≡N, N=N, O=O and N=O are 946, 418, 498 and 607 kJ mol^{-1} respectively. The resonance energy of N_2O is

(1) – 44 kJ
(2) – 62 kJ
(3) – 66 kJ
(4*) – 88 kJ

Sol. $N_2(g) + \frac{1}{2}O_2 \rightarrow N_2O(g)$

$\frac{1}{2}(O = O \rightarrow) \ddot{N} = N = \ddot{O}(g)$

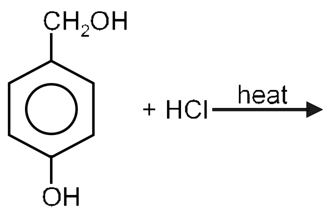
$\Delta H_f^0 =$ [Energy required for breaking of bonds]

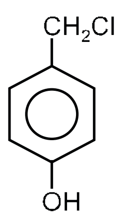
–[Energy released for forming of bonds]

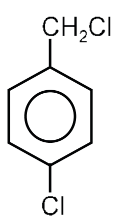
$= (\Delta H_{N\equiv N} + \frac{1}{2}\Delta H_{O=O} - \Delta H_{N=N} + \Delta H_{N=O})$

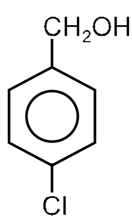
$= (946 + \frac{1}{2} \times 498) - (418 + 607) = 170 \text{kJ mol}^{-1}$

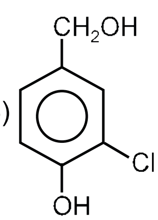
Resonance energy = 170 – 82 = 88 kJ mol^{-1}

52. The major product in the following reaction  is :

(1) 

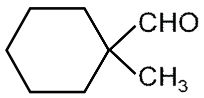
(2*) 

(3) 

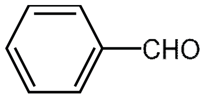
(4) 

53. Cannizaro's reaction is not given by :

(1*) CH_3CHO

(2) 

(3) HCHO

(4) 

Sol. Only those aldehydes which do not have α-H atom undergo Cannizaro's reaction. Hence CH_3CHO will not undergo Cannizaro's reaction as it has 3 α-H atoms.

54. Among the following vitamins the one whose deficiency causes rickets (bone deficiency) is :

(1) Vitamin A (2) Vitamin B (3) Vitamin C (4*) Vitamin D

Sol. Deficiency of vitamin D causes rickets.

55. The internuclear distances in O–O bonds for O_2^+, O_2 , O_2^- and O_2^{2-} respectively are :

(1) 1.30 Å, 1.49 Å, 1.12 Å, 1.21 Å

(2*) 1.12 Å, 1.21 Å, 1.30Å, 1.49Å

(3) 1.21Å, 1.12Å, 1.49Å, 1.30 Å

(4) 1.49Å, 1.21Å, 1.12Å, 1.30Å

Sol. The bond length follows the order

$$O_2^+ < O_2 < O_2^- < O_2^{2-}$$

According to this the possible values are 1.12 Å, 1.21 Å, 1.30Å, 1.49Å

56. The structure of which of the following chloro species can be explained on the basis of dsp^2 hybridisation?

(1) $CoCl_4^{2-}$ (2) $NiCl_4^{2-}$ (3*) $PdCl_4^{2-}$ (4) $FeCl_4^{2-}$

Sol. $[PdCl_4]^{2-}$ is dsp^2 hybridized and square planar in shape.

57. 6 litres of an alkene require 27 litres of oxygen at constant temperature and pressure for complete combustion. The alkene is :

(1*) 2-butene (2) Propene (3) 1-butene (4) Ethene

58. The ratio $\frac{K_p}{K_c}$ for the reaction, $CO\,(g) + \frac{1}{2}O_2\,(g) \rightleftharpoons CO_2\,(g)$ is :

(1) RT (2) $(RT)^{1/2}$ (3) 1 (4*) $\frac{1}{\sqrt{RT}}$

Sol. $K_P = K_C(RT)^{\Delta n_g}$

For the reaction

$$CO(g) + \frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$$

$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

$$\therefore K_P = \frac{K_C}{\sqrt{RT}}; \frac{K_P}{K_C} = \frac{1}{\sqrt{RT}}$$

59. In which of the following octahedral complex species the magnitude of Δ_0 will be maximum?

(1) $[Co(H_2O)_6]^{2+}$ (2) $[Co(NH_3)_6]^{3+}$ (3*) $[Co(CN)_6]^{3-}$ (4) $[Co(C_2O_4)_3]^{3-}$

60. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows:

$$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2,\ \Delta_rG = +940\ kJ\ mol^{-1}$$

The potential difference needed for the electrolytic reduction of aluminium oxide at 500°C should be atleast:

(1) 5.0 V (2) 2.5 V (3*) 3.0 V (4) 4.5 V

Sol. In the reaction

$$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2$$

For the oxidation half-reaction

$Al^{3+} + 3e^- \longrightarrow Al$

no. of electron transferred (n) = 3

$\Delta G^o = nFE^o$

$940 = 3 \times 96500 \times E^o$

$$E^o = \frac{940 \times 10^3\ J}{3 \times 96500}$$

$= 3.24 \approx 3$ V

PART-C-MATHEMATICS

61. If for positive integers r > 1, n > 2, the coefficients of the $(3r)^{th}$ and $(r + 2)^{th}$ powers of x in the expansion of $(1 + x)^{2n}$ are equal, then n is equal to :

(1*) 2r + 1 (2) 2r – 1 (3) 3r (4) r + 1

Sol. Expansion of $(1 + x)^{2n}$ is $1 + {}^{2n}C_1x + {}^{2n}C_2x^2 + \ldots\ldots + {}^{2n}C_rx^r + {}^{2n}C_{r+1}x^{r+1} + \ldots\ldots + {}^{2n}C_{2n}x^{2n}$

As given ${}^{2n}C_{r+2} = {}^{2n}C_{3r}$

$$\Rightarrow \frac{(2n)!}{(r+2)!(2n-r-2)!} = \frac{(2n)!}{(3r)!(2n-3r)!}$$

$\Rightarrow (3r)!\,(2n-3r)! = (r+2)!\,(2n-r-2)!$ ....(i)

RHS : (r + 2)! (3r)!

$\Rightarrow$ LHS = RHS

62. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 30°, then $|(\vec{a}\times\vec{b})\times\vec{c}|$ equals :

(1) $\frac{1}{2}$ (2*) $\frac{3}{2}$ (3) 3 (4) $\frac{3\sqrt{3}}{2}$

Sol. $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} + \hat{j}$

$\Rightarrow |\vec{a}| = 3$

and $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{4+4+1} = 3$

Now, $|\vec{c}-\vec{a}| = 2\sqrt{2} \Rightarrow |\vec{c}-\vec{a}|^2 = 8$

$\Rightarrow |\vec{c}-\vec{a}|\cdot(\vec{c}-\vec{a}) = 8$

$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}\cdot\vec{a} = 8$

$\Rightarrow |\vec{c}|^2 + 9 - 2|\vec{c}| = 8$

$\Rightarrow (|\vec{c}| - 1)^2 = 0 \Rightarrow |\vec{c}| = 1$

$$\therefore\ |(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}|\sin 30° = 3\times 1\times\frac{1}{2} = \frac{3}{2}$$

63. Let A = {θ : sin θ = tan θ} and B = {θ : cos θ = 1} be two sets. Then :

(1) A = B (2*) $A \not\subset B$ (3) $B \not\subset A$ (4) $A \subset B$ and $B - A \neq \phi$

Sol. Let A = {θ : sin θ = tan θ} and B = {θ : cos θ = 1}

Now, $A = \left\{\theta : \sin\theta = \frac{\sin\theta}{\cos\theta}\right\}$

= {θ : sin θ (cos θ – 1) = 0}

= {θ = 0, π, 2π, 3π,.......}

For B : cos θ = 1 $\Rightarrow$ $\theta = \pi, 2\pi, 4\pi, \ldots$

This shows that A is not contained in B. i.e. $A \not\subset B$. but $B \subset A$.

64. The value of $1^2 + 3^2 + 5^2 + \ldots\ldots + 25^2$ is :

(1*) 2925 (2) 1469 (3) 1728 (4) 1456

Sol. Consider $1^2 + 3^2 + 5^2 + \ldots + 25^2$

n^{th} term $T_n = (2n - 1)^2$, n = 1, ……. 13

Now, $S_n = \sum_{n=1}^{13} T_n = \sum_{n=1}^{13} (2n-1)^2$

$= \sum_{n=1}^{13} 4n^2 + \sum_{n=1}^{13} 1 - \sum_{n=1}^{13} 4n$

$= 4\sum n^2 + 13 - 4\sum n$

$= 4\left[\frac{n(n+1)(2n+1)}{6}\right] + 13 - 4\frac{n(n+1)}{2}$

put n = 13, we get

$S_n = 26 \times 14 \times 9 + 13 - 26 \times 14$

= 3276 + 13 – 364

= 2925

65. If an equation of a tangent to the curve y = cos (x + y), $-1 \le x \le 1 + \pi$, is x + 2y = k, then k is equal to :

(1) 1 (2) $\frac{\pi}{4}$ (3*) $\frac{\pi}{2}$ (4) 2

Sol. Let y = cos (x + y)

$\Rightarrow \frac{dy}{dx} = -\sin(x+y)\left(1+\frac{dy}{dx}\right)$ ….(i)

Now, given equation of tangent is

x + 2y = k

$\Rightarrow \text{Slope} = \frac{-1}{2}$

So, $\frac{dy}{dx} = \frac{-1}{2}$ put this value in (1), we get

$\frac{-1}{2} = -\sin(x+y)\left(1-\frac{1}{2}\right)$

$\Rightarrow$ sin (x + y) = 1

$\Rightarrow x + y = \frac{\pi}{2} \Rightarrow y = \frac{\pi}{2} - x$

Now, $\frac{\pi}{2} - x = \cos(x+y)$

$\Rightarrow x = \frac{\pi}{2} \text{and } y = 0$

Thus $x+2y=k \Rightarrow \frac{\pi}{2}=k$

66. A spherical balloon is being inflated at the rate of 35 cc/min. The rate of increase in the surface area (in cm^2/min.) of the balloon when its diameter is 14 cm, is :

(1) 100 (2) $10\sqrt{10}$ (3) $\sqrt{10}$ (4*) 10

Sol. Volume of sphere $V=\frac{4}{3}\pi r^3$

$$\frac{dV}{dt}=\frac{4}{3}\cdot\pi\cdot 3r^2\cdot\frac{dr}{dt}$$

$$35=4\pi r^2\cdot\frac{dr}{dt} \text{ or } \frac{dt}{dt}=\frac{35}{4\pi r^2} \qquad \ldots(1)$$

Surface area of sphere = S = $4\pi r^2$

$$\frac{dS}{dt}=4\pi\times 2r\times\frac{dt}{dt}=8\pi r\cdot\frac{dr}{dt}$$

$$\frac{dS}{dt}=\frac{70}{r} \qquad \text{(By using (1))}$$

Now, diameter = 14 cm, r = 7

$$\therefore \frac{dS}{dt}=10$$

67. If the integral $\int\frac{\cos 8x+1}{\cot 2x-\tan 2x}dx=A\cos 8x+k$, where k is an arbitrary constant, then A is equal to :

(1) $\frac{1}{8}$ (2) $\frac{1}{16}$ (3*) $\frac{-1}{16}$ (4) $\frac{-1}{8}$

Sol. Let $I=\int\frac{\cos 8x+1}{\cot 2x-\tan 2x}dx$

Now, $D'=\cot 2x-\tan 2x=\frac{\cos 2x}{\sin 2x}-\frac{\sin 2x}{\cos 2x}$

$$=\frac{\cos^2 2x-\sin^2 2x}{\sin 2x\cos 2x}=\frac{2\cos 4x}{\sin 4x}$$

$$\therefore I=\int\frac{2\cos^2 4x}{\frac{2\cos 4x}{\sin 4x}}dx=\int\frac{2\cos^2 4x.\sin 4x}{2\cos 4x}dx$$

$$=\frac{1}{2}\int\sin 8x dx=-\frac{1}{2}\frac{\cos 8x}{8}+k$$

$$=\frac{1}{16}.\cos 8x+k$$

$$\Rightarrow A=-\frac{1}{16}$$

68. Given a sequence of 4 numbers, first three of which are in G.P. and the last three are in A.P. with common difference six. If first and last terms of this sequence are equal, then the last term is :

(1) 16 (2) 4 (3) 2 (4*) 8

Sol. Let a, b, c, d be four numbers of the sequence.

Now, according to the question $b^2 = ac$ and $c - b = 6$ and $a - c = 6$

Also, given $\boxed{a = d}$

$$\therefore b^2 = ac \Rightarrow b^2 = a\left[\frac{a+b}{2}\right] \qquad (\because 2c = a + b)$$

$\Rightarrow a^2 - 2b^2 + ab = 0$

Now, $c - b = 6$ and $a - c = 6$,

gives $a - b = 12$

$\therefore a^2 - 2b^2 + ab = 0$

$\Rightarrow a^2 - 2(a - 12)^2 + a(a - 12) = 0$

$\Rightarrow a^2 - 2a^2 - 288 + 48a + a^2 - 12a = 0$

$\Rightarrow 36a = 288 \Rightarrow a = 8$

Hence, last term is $d = a = 8$.

69. A point on the ellipse, $4x^2 + 9y^2 = 36$, where the normal is parallel to the line, $4x - 2y - 5 = 0$ is :

(1) $\left(\frac{8}{5}, \frac{-9}{5}\right)$ (2*) $\left(\frac{-9}{5}, \frac{8}{5}\right)$ (3) $\left(\frac{9}{5}, \frac{8}{5}\right)$ (4) $\left(\frac{8}{5}, \frac{9}{5}\right)$

Sol. Given ellipse is $4x^2 + 9y^2 = 36$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$$

Normal at the point is parallel to the line $4x - 2y - 5 = 0$

Slope of normal = 2

Slope of tangent $= \frac{-1}{2}$

Point of contact to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and line is $\left(\frac{a^2m}{\sqrt{a^2m^2 + b^2}}, \frac{b}{\sqrt{a^2m^2 + b^2}}\right)$

Now, $a^2 = 9$, $b^2 = 4$

$\therefore$ Point $= \left(\frac{-9}{5}, \frac{8}{5}\right)$

70. If p and q are non-zero real numbers and $\alpha^3 + \beta^3 = -p$, $\alpha\beta = q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$ is :

(1) $px^2 + qx + p^2 = 0$ (2) $qx^2 - px + q^2 = 0$ (3*) $qx^2 + px + q^2 = 0$ (4) $px^2 - qx + p^2 = 0$

Sol. Given $\alpha^3 + \beta^3 = -p$ and $\alpha\beta = q$

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the root of required quadratic equation.

So, $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}=\frac{-p}{q}$

Hence, required quadratic equation is

$$x^2-\left(\frac{-p}{q}\right)x+q=0$$

$$\Rightarrow x^2+\frac{p}{q}x+q=0\Rightarrow qx^2+px+q^2=0$$

71. In a set of 2n observations, half of them are equal to 'a' and the remaining half are equal to '– a'. If the standard deviation of all the observations is 2; then the value of | a | is :

(1*) 2 (2) $\sqrt{2}$ (3) 2 (4) 4

Sol. Clearly mean A = 0

Now, standard deviation $\sigma=\sqrt{\frac{\sum(x-A)^2}{2n}}$

$$2=\sqrt{\frac{(a-0)^2+(a-0)^2+....+(0-a)^2+....}{2n}}$$

$$=\sqrt{\frac{a^2\cdot 2n}{2n}}=|a|$$

Hence, |a| = 2

72. Let p and q be any two logical statements and r : p → (~ p ∨ q). If r has a truth value F, then the truth values of p and q are respectively :

(1) F, F (2) T, T (3) F, T (4*) T, F

Sol. p → (~p ∨ q) has truth value F.

It means p → (~p ∨ q) is false.

It means p is true and ~p ∨ q is false.

⇒ p is true and both ~p and q are false.

⇒ p is true and q is false.

73. Let f : [– 2, 3] → [0, ∞) be a continuous function such that f (1 – x) = f(x) for all x ∈ [– 2, 3].

If R_1 is the numerical value of the area of the region bounded by y = f(x), x = – 2, x = 3 and the axis of x and $R_2=\int_{-2}^{3} x\,f(x)\,dx$, then :

(1*) $R_1 = 2R_2$ (2) $2R_1 = 3R_2$ (3) $R_1 = R_2$ (4) $3R_1 = 2R_2$

Sol. We have

$$R_2=\int_{-2}^{3}xf(x)dx=\int_{-2}^{3}(1-x)f(1-x)dx$$

$$\left[\text{Using}\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

$\Rightarrow R_2 = \int_{-2}^{3} (1-x)f(x)dx$ $\qquad$ ($\because$ f(x) = f(1 – x) on [–2, 3])

$\therefore R_2 + R_2 = \int_{-2}^{3} xf(x)dx + \int_{-2}^{3} (1-x)f(x)dx$

$= \int_{-2}^{3} f(x)dx = R_1$

$\Rightarrow 2R_1 = R_1$

74. Consider the function f(x) = [x] + | 1 – x |, – 1 ≤ x ≤ 3 where [x] is the greatest integer function.

Statement-1: f is not continuous at x = 0, 1, 2 and 3.

Statement-2: $f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1-x, & 0 \le x < 1 \\ 1+x, & 1 \le x < 2 \\ 2+x, & 2 \le x \le 3 \end{cases}$

(1) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(2) Statement-1 is false; Statement-2 is true.

(3*) Statement-1 is true; Statement-2 is false.

(4) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

Sol. Let f(x) = [x] + |1 – x|, –1 < x < 3

where [x] = greatest integer function.

f is not continuous at x = 0, 1, 2, 3

But in statement-2 f(x) is continuous at x = 3.

Hence, statement-1 is true and 2 is false.

75. If the events A and B are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$ and $P(B) = \frac{1-x}{4}$, then the set of possible values of x lies in the interval :

(1) [0, 1] $\qquad$ (2*) $\left[\frac{-1}{3}, \frac{5}{9}\right]$ $\qquad$ (3) $\left[\frac{-7}{9}, \frac{4}{9}\right]$ $\qquad$ (4) $\left[\frac{1}{3}, \frac{2}{3}\right]$

Sol. Since events A and B are mutually exclusive

$\therefore$ P(A) + P(B) = 1

$\Rightarrow \frac{3x+1}{3} + \frac{1-x}{4} = 1$

$\Rightarrow$ 12x + 4 + 3 – 3x = 12

$\Rightarrow 9x = 5 \Rightarrow x = \frac{5}{9}$

$\therefore x \in \left[\frac{1}{3}, \frac{5}{9}\right]$

76. For $0 \le x \le \frac{\pi}{2}$, the value of $\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t})\,dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t})\,dt$ equals :

(1) 1 (2) $\frac{-\pi}{4}$ (3) 0 (4*) $\frac{\pi}{4}$

Sol. Consider

$$\int_0^{\sin^2 x} \sin^{-1}\left(\sqrt{t}\right)dt + \int_0^{\cos^2 x} \cos^{-1}\left(\sqrt{t}\right)dt$$

Let I = f(x) after integrating and putting the limits.

$$f'(x) = \sin^{-1}\sqrt{\sin^2 x}(2\sin x\cos x) - 0 + \cos^{-1}\sqrt{\cos^2 x}(-2\cos x\sin x) - 0$$

$\therefore$ f '(x) = 0 $\Rightarrow$ f(x) = C (constant)

Now, we find f(x) at $x = \frac{\pi}{4}$

$$\therefore\ I = \int_0^{1/2} \sin^{-1}\sqrt{t}\,dt + \int_0^{1/2} \cos^{-1}\sqrt{t}\,dt$$

$$= \int_0^{1/2} (\sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t})dt$$

$$= \int_0^{1/2} \frac{\pi}{4}dt = \frac{\pi}{4} = C$$

$$\therefore\ f(x) = \frac{\pi}{4}$$

$\therefore$ Required integration $= \frac{\pi}{4}$

77. Let z satisfy | z | = 1 and $z = 1 - \bar{z}$.

Statement-1: z is a real number.

Statement-2: Principal argument of z is $\frac{\pi}{3}$.

(1) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(2) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(3*) Statement-1 is false; Statement-2 is true.

(4) Statement-1 is true; Statement-2 is false.

Sol. Let $z = x + iy, \bar{z} = x - iy$

Now, $z = 1 - \bar{z}$

$\Rightarrow x + iy = 1 - (x - iy)$

$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

Now, $|z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

$\Rightarrow y = \pm\frac{\sqrt{3}}{2}$

Now, $\tan\theta = \frac{y}{x}$ (θ is the argument)

$= \frac{\sqrt{3}}{2} \div \frac{1}{2}$ (+ve since only principal argument)

$= \sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Hence, z is not a real number

So, statement-1 is false and 2 is true.

78. **Statement-1:** The only circle having radius $\sqrt{10}$ and a diameter along line $2x + y = 5$ is $x^2 + y^2 - 6x + 2y = 0$.

Statement-2: $2x + y = 5$ is a normal to the circle $x^2 + y^2 - 6x + 2y = 0$.

(1*) Statement-1 is false; Statement-2 is true.

(2) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(3) Statement-1 is true; Statement-2 is false.

(4) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

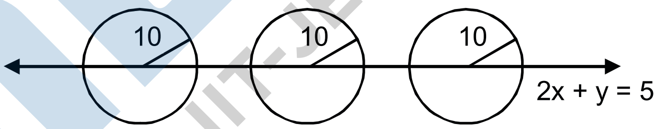
Sol. Circle : $x^2 + y^2 - 6x + 2y = 0$ ....(i)

Line : $2x + y = 5$ ....(ii)

Centre : (3, – 1)

Now, $2 \times 3 - 1 = 5$, hence centre lies on the given line. Therefore line passes through the centre. The given line is normal to the circle.

Thus statement-2 is true, but statement-1 is not true as there are infinite circle according to the given conditions.



79. Let $x \in (0, 1)$. The set of all x such that $\sin^{-1}x > \cos^{-1}x$, is the interval

(1) (0, 1) (2*) $\left(\frac{1}{\sqrt{2}}, 1\right)$ (3) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ (4) $\left(0, \frac{\sqrt{3}}{2}\right)$

Sol. Given $\sin^{-1} x > \cos^{-1} x$ where $x \in (0, 1)$

$\Rightarrow \sin^{-1} x > \frac{\pi}{2} - \sin^{-1} x$

$\Rightarrow 2\sin^{-1} x > \frac{\pi}{2} \Rightarrow \sin^{-1} x > \frac{\pi}{4}$

$\Rightarrow x > \sin\frac{\pi}{4} \Rightarrow x > \frac{1}{\sqrt{2}}$

Maximum value of $\sin^{-1} x$ is $\frac{\pi}{2}$

So, maximum value of x is 1. So, $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$.

80. The equation of a plane through the line of intersection of the planes x + 2y = 3, y – 2z + 1 = 0 and perpendicular to the first plane is :

(1) 2x – y – 10z = 9 (2) 2x – y – 9z = 10

(3*) 2x – y + 10z = 11 (4) 2x – y + 7z = 11

Sol. Equation of a plane through the line of intersection of the planes

x + 2y = 3, y – 2z + 1 = 0 is

$(x + 2y - 3) + \lambda(y - 2z + 1) = 0$

$\Rightarrow x + (2 + \lambda)y - 2\lambda(z) - 3 + \lambda = 0$ ....(i)

Now, plane (i) is $\perp$ to x + 2y = 3

$\therefore$ Their dot product is zero

i.e. $1 + 2(2+\lambda) = 0 \Rightarrow \lambda = \frac{5}{2}$

Thus, required plane is

$$x + \left(2 - \frac{5}{2}\right)y - 2 \times \frac{-5}{2}(z) - 3 - \frac{5}{2} = 0$$

$$\Rightarrow x - \frac{y}{2} + 5z - \frac{11}{2} = 0$$

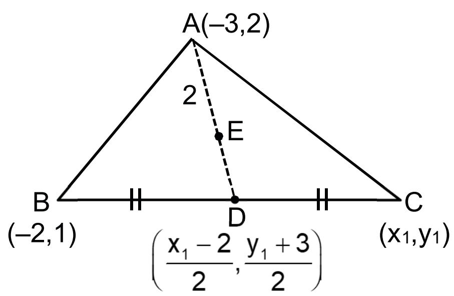
$\Rightarrow 2x - y + 10z - 11 = 0$

81. Let A (– 3, 2) and B (– 2, 1) be the vertices of a triangle ABC. If the centroid of this triangle lies on the line 3x + 4y + 2 = 0, then the vertex C lies on the line :

(1) 4x + 3y + 5 = 0 (2*) 3x + 4y + 3 = 0 (3) 3x + 4y + 5 = 0 (4) 4x + 3y + 3 = 0

Sol. Let C = (x_1, y_1)

Centroid, $E = \left(\frac{x_1 - 5}{3}, \frac{y_1 + 3}{3}\right)$



Since centroid lies on the line 3x + 4y + 2 = 0

$$\therefore 3\left(\frac{x_1 - 5}{3}\right) + 4\left(\frac{y_1 + 3}{3}\right) + 2 = 0$$

$\Rightarrow 3x_1 + 4y_1 + 3 = 0$

Hence vertex (x_1, y_1) lies on the line 3x + 4y + 3 = 0

82. 5-digit numbers are to be formed using 2, 3, 5, 7, 9 without repeating the digits. If p be the number of such numbers that exceed 20000 and q be the number of those that lie between 30000 and 90000, then p : q is :

(1) 6 : 5 (2) 3 : 2 (3*) 5 : 3 (4) 4 : 3

Sol. p :
0 0 0 0 0 place
5 4 3 2 1 ways

Total no. of ways = 5! = 120

Since all numbers are > 20, 000

$\therefore$ all numbers 2, 3, 5, 7, 9 can come at first place.

q :
0	0	0	0	0	place
3	4	3	2	1	ways

Total no. of ways = 3 × 4! = 72

($\because$ 2 and 9 can not be put at first place)

So, p : q = 120 : 72 = 5 : 3

83. The equation of the curve passing through the origin and satisfying the differential equation

$(1 + x^2)\,\dfrac{dy}{dx} + 2xy = 4x^2$ is :

(1*) $3(1 + x^2)\,y = 4x^3$ (2) $(1 + x^2)\,y = 3x^3$ (3) $3(1 + x^2)\,y = 2x^3$ (4) $(1 + x^2)\,y = x^3$

Sol. Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{4x^2}{1+x^2}$$

This is linear diff. equation

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$$

$\therefore$ Solution is

$$y(1+x^2) = \int \frac{4x^2}{1+x^2} \times 1 + x^2 + C$$

$$\Rightarrow y(1+x^2) = \frac{4x^3}{3} + C$$

$\Rightarrow$ Required curve is

$3y\,(1 + x^2) = 4x^3$ ($\because$ C = 0)

84. Let f(1) = – 2 and $f'(x) \geq 4.2$ for $1 \leq x \leq 6$. The possible value of f(6) lies in the interval :

(1*) $[19, \infty)$ (2) $[15, 19)$ (3) $(-\infty, 12)$ (4) $[12, 15)$

Sol. Given f(1) = – 2 and $f'(x) \geq 4.2$ for $1 \leq x \leq 6$

Consider $f'(x) = \dfrac{(x+h) - f(x)}{h}$

$\Rightarrow$ f(x + h) – f(x) = f '(x) . h $\geq$ (4.2)h

So, f(x + h) $\geq$ f(x) + (4.2)h

put x = 1 and h = 5, we get

f(6) $\geq$ f(1) + 5(4.2) $\Rightarrow$ f(6) $\geq$ 19

Hence f(6) lies in $[19, \infty)$

85. Let $S = \left\{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix} : a_{ij} \in [0, 1, 2], a_{11} = a_{22}\right\}$. Then the number of non-singular matrices in the set S is :

(1) 10 (2) 24 (3*) 20 (4) 27

Sol. The matrices in the form

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, a_{ij} \in \{0,1,2\}, a_{11} = a_{12}$ are

$\begin{bmatrix} 0 & 0/1/2 \\ 0/1/2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0/1/2 \\ 0/1/2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0/1/2 \\ 0/1/2 & 2 \end{bmatrix}$

At any place, 0/1/2 means 0, 1 or 2 will be the element at that place.

Hence there are total 27 = 3 × 3 + 3 × 3 + 3 × 3 matrices of the above form. Out of which the matrices which are singular are

$\begin{bmatrix} 0 & 0/1/2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1/2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Hence there are total 7(= 3 + 2 + 1 + 1) singular matrices.

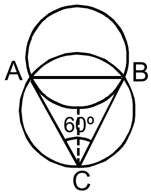
Therefore number of all non-singular matrices in the given form = 27 – 7 = 20

86. If a circle of unit radius is divided into two parts by an arc of another circle subtending an angle 60° on the circumference of the first circle, then the minimum diameter of the circle of that arc is :

(1) $\frac{1}{2}$ (2) 1 (3*) $\sqrt{3}$ (4) $\sqrt{2}$

Sol. Solving we get $AB = \sqrt{3}$

$\therefore$ diameter (minimum) $= \sqrt{3}$



87. If the image of point P (2, 3) in a line L is Q (4, 5), then the image of point R (0, 0) in the same line is :

(1) (4, 5) (2) (3, 4) (3) (2, 2) (4*) (7, 7)

Sol. Mid-Point of P(2, 3) and Q(4, 5) = (3, 4)

Slope of PQ = 1

Slope of the line L = –1

Mid-point (3, 4) lies on the line L.

Equation of line L.

$y - 4 = -1(x - 3) \Rightarrow x + y - 7 = 0$ ....(i)

Mid-point of RS $= \left(\frac{x_1}{2}, \frac{y_1}{2}\right)$

Mid-point $\left(\frac{x_1}{2}, \frac{y_1}{2}\right)$ line on the line (i)

$\therefore\ x_1 + y_1 = 14$

Slope of RS $= \frac{y_1}{x_1}$

Since RS $\perp$ line L

$\therefore \frac{y_1}{x_1} \times (-1) = -1$

$\therefore x_1 = y_1$ ....(ii)

From (ii) and (iii)

$x_1 = y_1 = 7$

Hence the image of R = (7, 7)

88. Consider the system of equations: x + ay = 0, y + az = 0 and z + ax = 0. Then the set of all real values of 'a' for which the system has a unique solution is :

(1) {1, – 1} (2*) R – {– 1} (3) {1, 0, – 1} (4) R – {1}

Sol. Given system of equations is homogeneous which is

x + ay = 0

y + az = 0

z + ax = 0

It can be written in matrix form as

$$A = \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix}$$

Now, $|A| = [1 - a(-a^2)] = 1 + a^3 \neq 0$

So, system has only trivial solution.

Now, |A| = 0 only when a = –1

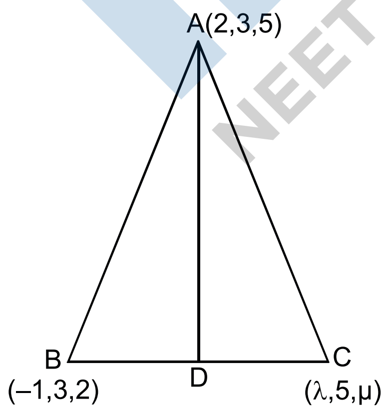
So, system of equations has infinitely many solutions which is not possible because it is given that system has a unique solution. Hence set of all real values of 'a' is

R – {– 1}.

89. Let ABC be a triangle with vertices at points A (2, 3, 5), B (– 1, 3, 2) and C (λ, 5, μ) in three dimensional space. If the median through A is equally inclined with the axes, then (λ, μ) is equal to :

(1) (7, 5) (2) (10, 7) (3) (5, 7) (4*) (7, 10)

Sol. Since AD is the median



$$\therefore D = \left(\frac{\lambda - 1}{2}, 4, \frac{\mu + 2}{2}\right)$$

Now, dR's of AD is

$$a = \left(\frac{\lambda - 1}{2} - 2\right) = \frac{\lambda - 5}{2}$$

$$b = 4 - 3 = 1, c = \frac{\mu + 2}{2} - 5 = \frac{\mu - 8}{2}$$

Also, a, b, c are dR's

$\therefore$ a = kℓ, b = km, c = kn where ℓ = m = n

and $\ell^2 + m^2 + n^2 = 1$

$$\Rightarrow \ell = m = n = \frac{1}{\sqrt{3}}$$

Now, a = 1, b = 1 and c = 1

$\Rightarrow \lambda = 7$ and $\mu = 10$

90. A common tangent to the conics $x^2 = 6y$ and $2x^2 - 4y^2 = 9$ is :

(1) x + y = 1 (2) x – y = 1 (3) $x + y = \frac{9}{2}$ (4*) $x - y = \frac{3}{2}$

Sol. $x^2 - 6y$ ....(i)

$2x^2 - 4y^2 = 9$ ....(ii)

$x - y = \frac{3}{2}$ ....(iii)

On solving (i) and (iii), we get only

$$x = 3, y = \frac{3}{2}$$

Hence $\left(3, \frac{3}{2}\right)$ is the point of contact of conic (i), and line (iii)

Hence line (iii) is the common tangent to both the given conics.